6.4 Binomial Coefficients and Identities

Problem

Find the coefficient of the term for when the power of a is 17 in $(2a + 3b)^{23}$.

Recall the binomial theorem:

$$(x+y)^n = \sum_{j=0}^n \binom{n}{j} x^{n-j} y^j.$$

Here, x = 2a, y = 3b, and n = 23. Now consider the appropriate term:

$$\binom{23}{6}(2a)^{17}(3b)^6.$$

Then the coefficient will be:

$$2^{17} \cdot 3^6 \cdot C(23,6) = 2^{17} \cdot 3^6 \cdot \frac{23!}{6!17!}.$$