6.2 The Pigeonhole Principle

6.2 pg 405 # 1

Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

There are six classes (pigeons), but only five weekdays (pigeonholes). Therefore, by the pigeonhole principle, at least $\lceil \frac{6}{5} \rceil = 2$ classes must be held on the same day.

6.2 pg 405 # 5

Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

There are four possible remainders when an integer is divided by 4: 0, 1, 2, or 3 (these are pigeonholes). Therefore, by the pigeonhole principle at least two $(= \lceil \frac{5}{4} \rceil)$ of the five given remainders (these are pigeons) must be the same.

6.2 pg 406 # 35

There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?

The 38 time periods are the pigeonholes, and the 677 classes are the pigeons. By the generalized pigeonhole principle there is at least one time period in which at least $\lceil \frac{677}{38} \rceil = 18$ classes are meeting. Since each class must meet in a different room, we need 18 rooms.