1.5 Nested Quantifiers

Nested quantifiers are quantifiers that occur within the scope of other quantifiers.
Example: $\forall x \exists y P(x, y)$

Quantifier order matters!
$\forall x \exists y P(x, y) \neq \exists y \forall x P(x, y)$

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Let $L(x, y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry.
   $\forall x L(x, Jerry)$

b) Everybody loves somebody.
   $\forall x \exists y L(x, y)$

c) There is somebody whom everybody loves.
   $\exists y \forall x L(x, y)$

d) Nobody loves everybody.
   $\forall x \exists y \neg L(x, y)$ or $\neg \exists x \forall y L(x, y)$

i Everyone loves himself or herself
   $\forall x L(x, x)$

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Let $W(x, y)$ mean that student x has visited website y, where the domain for x consists of all students in your school and the domain for y consists of all websites. Express each of these statements by a simple English sentence.

d $\exists y(W(\text{Ashok Puri}, y) \wedge W(\text{Cindy Yoon}, y))$
   There is a website that both Ashok and Cindy have visited.

e $\exists y \forall z(y \neq (\text{David Belcher}) \wedge (W(\text{David Belcher}, z) \rightarrow W(y, z)))$
   There is a person besides David who has visited all the websites that David has visited.

f $\exists x \exists y \forall z(((x \neq y) \wedge (W(x, z) \leftrightarrow W(y, z))))$
   There are two distinct people who have visited exactly the same sites.

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Let $M(x, y)$ be "x has sent y an e-mail message" and $T(x, y)$ be "x has telephoned y," where the domain consists for all students in your class. Use quantifiers to express each of these statements.

k There is a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class.

$\exists x \forall y ((x \neq y) \rightarrow (\neg M(y, x) \wedge \neg T(y, x)))$

l Every student in the class has either received an e-mail message or received a telephone call from another student in the class.

$\forall x \exists y ((x \neq y) \wedge (M(y, x) \vee T(y, x)))$

m There are at least two students in your class such that one student has sent the other e-mail and the second student has telephoned the first student

$\exists x \exists y ((x \neq y) \wedge (M(x, y) \wedge T(y, x)))$

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Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a) $\neg \forall x \forall y P(x, y)$

$$\begin{array}{ll} \neg \forall x \forall y P(x, y) & \\ \equiv \exists x \neg \forall y P(x, y) & \text{De Morgan's laws for quantifiers} \\ \equiv \exists x \exists y \neg P(x, y) & \text{De Morgan's laws for quantifiers} \end{array}$$

d $\neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y (Q(x, y))$

$$\begin{array}{ll} \neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y (Q(x, y)) & \\ \equiv (\neg \exists x \exists y \neg P(x, y)) \vee (\neg \forall x \forall y Q(x, y)) & \text{De Morgan's Law} \\ \equiv (\forall x \neg \exists y \neg P(x, y)) \vee (\neg \forall x \forall y Q(x, y)) & \text{De Morgan's laws for quantifiers} \\ \equiv (\forall x \forall y \neg \neg P(x, y)) \vee (\neg \forall x \forall y Q(x, y)) & \text{De Morgan's laws for quantifiers} \\ \equiv (\forall x \forall y P(x, y)) \vee (\neg \forall x \forall y Q(x, y)) & \text{Double Negation} \\ \equiv (\forall x \forall y P(x, y)) \vee (\exists x \neg \forall y Q(x, y)) & \text{De Morgan's laws for quantifiers} \\ \equiv (\forall x \forall y P(x, y)) \vee (\exists x \exists y \neg Q(x, y)) & \text{De Morgan's laws for quantifiers} \end{array}$$