1.4 Predicates and Quantifiers

Predicate Logic

Predicate logic have the following features to express propositions:

- Variables: x, y, z, etc. (the subject of a sentence), can be substituted with an element from a domain.
- Predicates: P, M, etc. (the predicate of a sentence)
- Domain: the collection of values that a variable can take.

Propositions must be definitive (not vague or undefined). So, a Propositional Function is not a Proposition until all variables are defined (or "**bound**").

Example: Let $Q(x, y)$ denote the statement "$x + y > 2x$"

- $Q(x, y)$ has two unbound variables (x and y), and is not a proposition.
- $Q(1, y) = 1 + y > 2$ [Not a proposition] one bound variable ($x = 1$) and one unbound variable (y).
- $Q(1, 2) = 1 + 2 > 2$ [Proposition] two bound variables ($x = 1$ and $y = 2$)

Quantifiers

Quantifiers provide a notation that allows us to quantify (count) how many objects in the universe of discourse satisfy the given predicate.

- Universal Quantifier $\forall$ - For all elements
- Existential Quantifier $\exists$ - There exists an element

De Morgan's Law for Quantifiers

- $\neg\forall x P(x) \equiv \exists x \neg P(x)$
- $\neg\exists x P(x) \equiv \forall x \neg P(x)$

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Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$

   There exists a student who spends more than five hours every weekday in class.

b) $\forall x P(x)$

Every student spends more than five hours every weekday in class.

c) $\exists x \neg P(x)$

There exists a student who does not spend more than five hours every weekday in class.

d) $\forall x \neg P(x)$

No student spends more than five hours every weekday in class.

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Let $P(x)$ be the statement "$x = x^2$." If the domain consists of all the integers, what are these truth values?

a) $P(0)$

True

c $P(2)$

False

e $\exists x P(x)$

True

f $\forall x P(x)$

False

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Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. The domain of x is all people.

c All your friends are perfect.

Let $F(x)$ be "x is your friend" and $P(x)$ be "x is perfect."
$\forall x(F(x) \to P(x))$

d At least one of your friends is perfect.

Let $F(x)$ be "x is your friend" and $P(x)$ be "x is perfect."
$\exists x(F(x) \land P(x))$

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Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

b No rabbit knows calculus.

$C(x) =$"x knows calculus".
Domain for x is all rabbits.
$\forall x \neg C(x)$
$\exists x C(x)$
There exists a rabbit that knows calculus.

c Every bird can fly.

$F(x) =$"x can fly".
Domain for x is all birds.
$\forall x F(x)$
$\exists x \neg F(x)$
There exists a bird who cannot fly.

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Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a $\forall x(x^2 \geq x)$

No counter example. See Example 13 in section 1.4.

b $\forall x(x > 0 \vee x < 0)$

0. Since 0 is not less than or greater than 0, 0 is a counter example.

c $\forall x(x = 1)$

2. Since 2 is not 1, 2 is a counter example.

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Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people.

a No professors are ignorant.

$\forall x(P(x) \rightarrow \neg Q(x))$

b All ignorant people are vain.

$\forall x(Q(x) \rightarrow R(x))$

c No professors are vain.

$\forall x(P(x) \to \neg R(x))$

d Does (c) follow from (a) and (b)?

No because we can have vain professors.