

7.3 Bayes' Theorem

Problem

Two friends want to go hiking on a Wednesday. Unfortunately, the local meteorologist has predicted rain on Wednesday. Suppose the probability that it will rain on a given day is 5%. If it rains, the meteorologist accurately predicts rain 99% of the time. If it doesn't rain, she inaccurately predicts rain 3% of the time. What is the probability that it will rain on Wednesday? (*Hint: use Bayes' theorem*).

For a given day, let A be the event "it rained for the given day," and let B be the event "the meteorologist predicted rain for the given day." Then we're given $p(A) = 0.05$, $p(B|A) = 0.99$, and $p(B|\bar{A}) = 0.03$. We're asked to find $p(A|B)$. We can use Bayes' theorem to accomplish this, which states:

$$p(A|B) = \frac{p(B|A)p(A)}{p(B|A)p(A) + p(B|\bar{A})p(\bar{A})}.$$

Then all we have to do is simply substitute the values we've found. Notice that $p(\bar{A}) = 1 - p(A) = 1 - 0.05 = 0.95$.

$$\begin{aligned} p(A|B) &= \frac{0.99 \cdot 0.05}{0.99 \cdot 0.05 + 0.03 \cdot 0.95} \\ &\approx 0.6346 \end{aligned}$$

Notice that even though the meteorologist is remarkably accurate, the chance that s/he's right is still not as high as you might expect if you were to take a guess.