7.1 An Introduction to Probability

Finite Probability

If S is a finite nonempty sample space of equally likely outcomes, and E is an event, that is, a subset of S, then the *probability* of E is $p(E) = \dfrac{|E|}{|S|}$.

Probabilities of Complements

Let E be an event in a sample space S. The probability of the event $\overline{E} = S - E$, the complementary event of E, is given by $p(\overline{E}) = 1 - p(E)$.

Probabilities of Unions of Events

Let E_1 and E_2 be events in the sample space S. Then

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

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What is the probability that when a coin is flipped six times in a row, it lands heads up every time?

Use $p(E) = |E|/|S|$.
$(1/2)^6 = 1/64$

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What is the probability that a five-card poker hand contains two pairs (that is, two of each of two different kinds and a fifth card of a third kind)?

In total, there are $C(52, 5)$ ways to draw a hand (this is our $|S|$).

We want to choose 2 out of four cards of one value, 2 out of four cards of another value, and one other card not of the first two values (This will be our $|E|$).

First we choose two values, there are 13 values (2 to A), so $C(13, 2)$.
Then we want to choose two cards of the first value out of four cards, $C(4, 2)$
Again, we want to choose two cards of the second value out of four cards, $C(4, 2)$
And finally, choose one card not of the previously selected types (we can't choose the 4 cards of the first value and the 4 cards of the second value), $C(52 - 8, 1)$

So we get:
$$\frac{C(13,2) \cdot C(4,2) \cdot C(4,2) \cdot C(44,1)}{C(52,5)}$$
$$= \frac{13!/(2!11!) \cdot 4!/(2!2!) \cdot 4!/(2!2!) \cdot 44!/(1!43!)}{52!/(47!5!)}$$
$= 198/4165 \approx 0.0475$

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What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?

Use $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

$p(E_1)$ represents the probability of a number being divisible by 5.
$p(E_2)$ represents the probability of a number being divisible by 7.
$p(E_1 \cap E_2)$ represents the probability of a number being divisible by 35 (divisible by both 5 and 7)

There are:
$\lfloor 100/5 \rfloor = 20$ positive integers divisible by 5.
$\lfloor 100/7 \rfloor = 14$ positive integers divisible by 7.
$\lfloor 100/35 \rfloor = 2$ positive integers divisible by 35.

$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$
$= \frac{20}{100} + \frac{14}{100} - \frac{2}{100}$
$= \frac{32}{100}$

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What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 200 people enter a contest and

a) no one can win more than one prize.

   p(Abbey winning first) = $\frac{1}{200}$
   p(Barry winning second) = $\frac{1}{199}$
   p(Sylvia winning third) = $\frac{1}{198}$

   $\frac{1}{200} \cdot \frac{1}{199} \cdot \frac{1}{198}$
   $= \frac{1}{7880400}$

b) winning more than one prize is allowed.

   p(Abbey winning first) = $\frac{1}{200}$
   p(Barry winning second) = $\frac{1}{200}$
   p(Sylvia winning third) = $\frac{1}{200}$

$$\frac{1}{200} \cdot \frac{1}{200} \cdot \frac{1}{200}$$
$$= \frac{1}{8000000}$$