

1.1 Propositional Logic

Proposition

Definition: A proposition (denoted p, q, r, \dots)

- A statement (not an order or a question)
- Definitive (not vague or undefined)
- Either True or False, but not both at the same time
- Truth value might be known or conditional

Proposition Logic

Definition: Propositional Logic deals with statements (propositions) and compound statements built from simpler statements using logical connectives.

Negation

p	$\neg p$
T	F
F	T

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (Inclusive OR)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse, Contrapositive, Inverse

- The **converse** of $p \rightarrow q$ is $q \rightarrow p$
- The **contrapositive** of $p \rightarrow q$ is $\neg q \rightarrow \neg p$
- The **inverse** of $p \rightarrow q$ is $\neg p \rightarrow \neg q$

Precedence of Logical Operators

Operator	Precedence
\neg	1
\wedge	2
\vee	3
\rightarrow	4
\leftrightarrow	5

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Let p and q be the propositions

- p : You drive over 65 miles per hour.
- q : You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

- b) You drive over 65 miles per hour, but you do not get a speeding ticket.

$$p \wedge \neg q$$

- c) You will get a speeding ticket if you drive over 65 miles per hour.

$$p \rightarrow q$$

- f) You get a speeding ticket, but you do not drive over 65 miles per hour. $q \wedge \neg p$

- g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

$$q \rightarrow p$$

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Write each of these statements in the form “if p , then q ” in English. [Hint: Refer to the list of common ways to express conditional statements.]

- a) It snows whenever the wind blows from the northeast.
If the wind blows from the northeast, then it snows.
- b) The apple trees will bloom if it stays warm for a week.
If it stays warm for a week, then the apple trees will bloom.
- c) That the Pistons win the championship implies that they beat the Lakers.
If the Pistons win the championship, then they beat the Lakers.
- d) It is necessary to walk 8 miles to get to the top of Long’s Peak.
If you get to the top of Long’s Peak, then you walked 8 miles.

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Write each of these propositions in the form “ p if and only if q ” in English.

- a If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone, it is hot outside.

You buy an ice cream cone if and only if it is hot outside.

- b For you to win the contest it is necessary and sufficient that you have the only winning ticket.

You win the contest if and only if you have the only winning ticket.

- c You get promoted only if you have connections, and you have connections only if you get promoted.

You get promoted if and only if you have connections.

- d If you watch television your mind will decay, and conversely.

Your mind will decay if and only if you watch television.

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Construct a truth table for each of these compound propositions

a) $(p \vee q) \rightarrow (p \oplus q)$

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

c) $(p \vee q) \oplus (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

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Construct a truth table for each of these compound propositions

a) $p \rightarrow (\neg q \vee r)$

p	q	r	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

b) $\neg p \rightarrow (q \rightarrow r)$

p	q	r	$\neg p$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

p	q	r	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow r$	$(p \rightarrow q) \wedge (\neg p \rightarrow r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Bit Operations

A **bit string** is a sequence of zero or more bits. The length of this string is the number of bits in the string.

The bitwise OR, bitwise AND, and bitwise XOR of two strings of the same length to be the strings that have as their bits the OR, AND, and XOR of the corresponding bits in the two strings, respectively. We use the symbols \vee , \wedge , and \oplus to represent the bitwise OR, bitwise AND, and bitwise XOR operations, respectively.

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Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

a) 101 1110, 010 0001

OR: 111 1111

AND: 000 0000

XOR 111 1111

b) 1111 0000, 1010 1010

OR: 1111 1010

AND: 1010 0000

XOR: 0101 1010

c) 00 0111 0001, 10 0100 1000

OR: 10 0111 1001

AND: 00 0100 0000

XOR: 10 0011 1001

d) 11 1111 1111, 000 0000 0000

OR: 11 1111 1111

AND: 00 0000 0000

XOR: 11 1111 1111