### 13.2 Finite-State Machines with Output

A finite-state machine $M=\left(S, I, O, f, g, s_{0}\right)$ consists of

- a finite set $S$ of states
- a finite input alphabet I
- a finite output alphabet $O$
- a transition function $f(f: S \times I \rightarrow S)$
- an output function $g(g: S \times I \rightarrow O)$
- an initial state $s_{0}$


| State | Input |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $s_{0}$ | $s_{0}, 1$ | $s_{1}, 0$ |
| $s_{1}$ | $s_{0}, 1$ | $s_{2}, 1$ |
| $s_{2}$ | $s_{2}, 1$ | $s_{1}, 0$ |

## Types of Finite-State Machines

- Mealy machines: outputs correspond to transitions between states
- Moore machine: output is determined only by the state



## Language Recognizer

Let $M=\left(S, I, O, f, g, s_{0}\right)$ be a finite-state machine and $L \subseteq I^{*}$. We say that $M$ recognizes (or accepts) $L$ if an input string $x$ belongs to $L$ if and only if the last output bit produced by $M$ when given $x$ as input is a 1 .

## 13.2 pg. 863 \# 1

Draw the state diagrams for the finite-state machines with these state tables.
a )

| State | Input |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $s_{0}$ | $s_{1}, 0$ | $s_{0}, 1$ |
| $s_{1}$ | $s_{0}, 0$ | $s_{2}, 1$ |
| $s_{2}$ | $s_{1}, 0$ | $s_{1}, 0$ |


b )

| State | Input |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $s_{0}$ | $s_{1}, 0$ | $s_{0}, 0$ |
| $s_{1}$ | $s_{2}, 1$ | $s_{0}, 1$ |
| $s_{2}$ | $s_{0}, 0$ | $s_{3}, 1$ |
| $s_{3}$ | $s_{1}, 1$ | $s_{2}, 0$ |



## 13.2 pg. 863 \# 3

Find the output generated from the input string 01110 for the finite-state machine with the state table in
a) Exercise 1(a).

The state transition sequence is:
$s_{0} \rightarrow s_{1} \rightarrow s_{2} \rightarrow s_{1} \rightarrow s_{2} \rightarrow s_{1}$
Our output is: 01010
b) Exercise 1(b).

The state transition sequence is:
$s_{0} \rightarrow s_{1} \rightarrow s_{0} \rightarrow s_{0} \rightarrow s_{0} \rightarrow s_{1}$
Our output is: 01000

## Lecture Notes 25 Exercise

Construct a finite-state machine with output that produces a 1 if and only if the last 3 input bits read are 0s.

| State | Input |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $s_{0}$ | $s_{1}, 0$ | $s_{0}, 0$ |
| $s_{1}$ | $s_{2}, 0$ | $s_{0}, 0$ |
| $s_{2}$ | $s_{2}, 1$ | $s_{0}, 0$ |



## 13.2 pg. 864 \# 9

Construct a finite-state machine that delays an input string two bits, giving 00 as the first two bits of output.

$s_{0}$ corresponds to the last two bits having been $00, s_{1}$ corresponds to the last two bits having been $01, s_{2}$ corresponds to the last two bits having been $10, s_{3}$ corresponds to the last two bits having been 11 .

