13.2 Finite-State Machines with Output

A finite-state machine $M = (S, I, O, f, g, s_0)$ consists of

- a finite set S of states
- a finite input alphabet I
- a finite *output alphabet* O
- a transition function $f(f: S \times I \to S)$
- an output function $g(g: S \times I \rightarrow O)$
- an *initial state* s_0



State	Inp	out
	0	1
s_0	$s_0, 1$	$s_1, 0$
s_1	$s_0, 1$	$s_2, 1$
s_2	<i>s</i> ₂ , 1	$s_1, 0$

Types of Finite-State Machines

- Mealy machines: outputs correspond to transitions between states
- *Moore machine:* output is determined only by the state



Language Recognizer

Let $M = (S, I, O, f, g, s_0)$ be a finite-state machine and $L \subseteq I^*$. We say that *M* recognizes (or *accepts*) *L* if an input string *x* belongs to *L* if and only if the last output bit produced by *M* when given *x* as input is a 1.

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Draw the state diagrams for the finite-state machines with these state tables.

 $s_1, 1 \mid s_2, 0$

a)

b)

State	Input		1,1
	0	1	
s_0	<i>s</i> ₁ , 0	<i>s</i> ₀ , 1	
s_1	$s_0, 0$	$s_2, 1$	start \rightarrow $s_0 \rightarrow$ $s_1 \rightarrow$ s_2
s_2	$s_1, 0$	$s_1, 0$	$ \bigcirc \overline{0,0} \overline{0,0} \overline{0,0} $
State	Inj	put	1,0
State	Inj 0	put	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
State	Inj 0 $s_1, 0$	$\frac{1}{s_0, 0}$	1,0 $1,1$ $1,1$ $1,0$ $0,0$ $0,1$ $1,1$
State	Inj 0 $s_1, 0$ $s_2, 1$	$ \begin{array}{c} 0 \\ 1 \\ s_0, 0 \\ s_0, 1 \end{array} $	start \rightarrow s_0 s_1 s_1 s_2 s_2 s_1 s_2 s_2 s_3

0.0

0,1

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Find the output generated from the input string 01110 for the finite-state machine with the state table in

a) Exercise 1(a).

The state transition sequence is: $s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow s_1 \rightarrow s_2 \rightarrow s_1$ Our output is: 01010

 s_3

b) Exercise 1(b).

The state transition sequence is: $s_0 \rightarrow s_1 \rightarrow s_0 \rightarrow s_0 \rightarrow s_0 \rightarrow s_1$ Our output is: 01000

Lecture Notes 25 Exercise

Construct a finite-state machine with output that produces a 1 if and only if the last 3 input bits read are 0s.



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Construct a finite-state machine that delays an input string two bits, giving 00 as the first two bits of output.



 s_0 corresponds to the last two bits having been 00, s_1 corresponds to the last two bits having been 01, s_2 corresponds to the last two bits having been 10, s_3 corresponds to the last two bits having been 11.