13.4 Language Recognition

Regular Sets

Sets that can be built up from concatenation, union, and Kleene closure in arbitrary order with the empty set, empty string, and singleton sets that can also be recognized by a finite-state automaton are called *regular sets*.

Regular Expressions

The *regular expressions* over a set I are defined recursively by:

- the symbol $\emptyset$ is a regular expression
- the symbol λ is a regular expression
- the symbol x is a regular expression whenever $x \in I$
- the symbols $(\mathbf{AB})$, $(\mathbf{A} \cup \mathbf{B})$ and $\mathbf{A}^*$ are regular expressions whenever $\mathbf{A}$ and $\mathbf{B}$ are regular expressions

Kleene's Theorem

A set is regular if and only if it is recognized by a finite-state automaton.

Theorem 2

A set is generated by a regular grammar if and only if it is a regular set.

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Determine whether 0101 belongs to each of these regular sets.

a) 01^*0^*

No.

b) $0(11)^*(01)^*$

No.

c) $0(10)^*1^*$

Yes.

d) $0^*10(0 \cup 1)$

Yes.

e) $(10)^*(11)^*$

No.

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Express each of these sets using a regular expression.

a) the set consisting of the strings 0, 11, and 010

$0 \cup 11 \cup 010$

b) the set of strings of three 0s followed by two or more 0s

000000^*

c) the set of strings of odd length

$(0 \cup 1)(00 \cup 01 \cup 10 \cup 11)^*$

d) the set of strings that contain exactly one 1

0^*10^*

e) the set of strings ending in 1 and not containing 000

$(1 \cup 01 \cup 001)^*$

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Express each of these sets using a regular expression.

a) the set of strings of one or more 0s followed by a 1

00^*1

b) the set of strings of two or more symbols followed by three or more 0s

$(0 \cup 1)(0 \cup 1)(0 \cup 1)^*0000^*$

c) the set of strings with either no 1 preceding a 0 or no 0 preceding a 1

$(0^*1^*) \cup (1^*0^*)$

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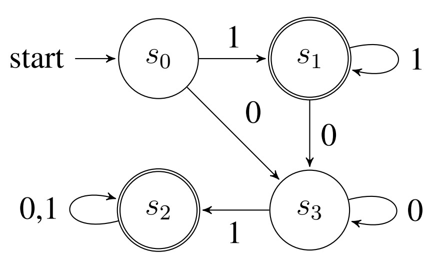
Construct a regular grammar $G = (V, T, S, P)$ that generates the language recognized by the given finite-state machine.



Our grammar can be defined by letting $V = \{0, 1, S, A, B, C\}$, $T = \{0, 1\}$, and the set of production rules P consisting of $S \to 0C, S \to 1A, A \to 0C, A \to 1A, C \to 0C, C \to 1B, B \to 0B, B \to 1B, S \to 1, A \to 1, B \to 0, B \to 1, C \to 1$.
S represents s_0, A represents s_1, B represents s_2, and C represents s_3.