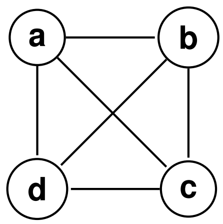
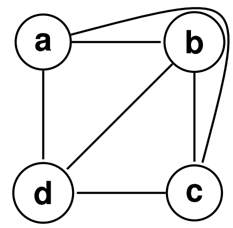
10.7 Planar Graphs

A graph is called *planar* if it can be drawn in the plane without any edges crossing (where a crossing of edges is the intersection of the lines or arcs representing them at a point other than their common endpoint)



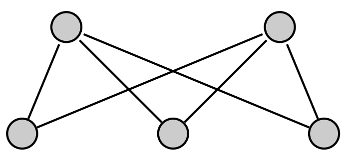


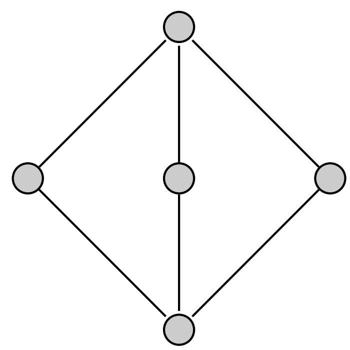
Euler's Formula

Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then $r = e - v + 2$.

10.7 pg. 725 # 3

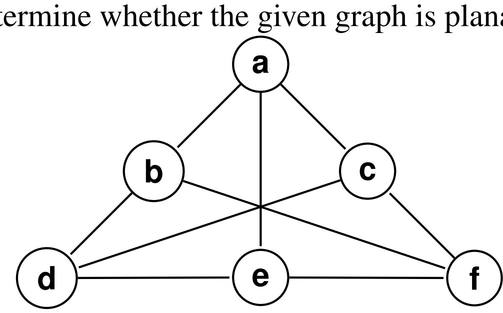
Draw the given planar graph without any crossings.





10.7 pg. 725 # 5

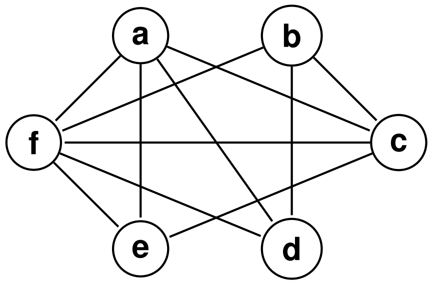
Determine whether the given graph is planar. If so, draw it so that no edges cross.

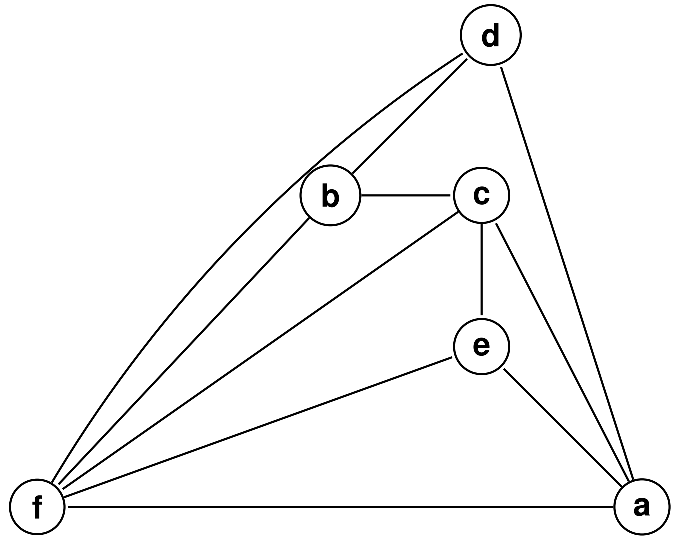


This graph is not planar because we can form a $K_{3,3}$ graph with the vertices $\{a, d, f\}$ and $\{b, c, e\}$

10.7 pg. 725 # 7

Determine whether the given graph is planar. If so, draw it so that no edges cross.





10.7 pg. 725 # 13

Suppose that a connected planar graph has six vertices, each of degree four. Into how many regions is the plane divided by a planar representation of this graph?

We apply Euler's formula where $r = e - v + 2$.
Since each vertex has degree 4, the sum of the degrees is 24. By the handshaking theorem, $2e = 24$ so $e = 12$.
$r = 12 - 6 + 2$
$r = 8$

Thus we have 8 regions in this planar graph.