10.6 Shortest-Path Problems

- Given a graph $G = (V, E)$, a weighting function $w(e), w(e) > 0$, for the edges of G, and a source vertex, v_0.
- We wish to determine a shortest path from v_0 to v_n

Dijkstra's Algorithm

Dijkstra's algorithm is a common algorithm used to determine shortest path from a to z in a graph.

Algorithm $dijkstra(G:$ weighted connected simple graph with all weights positive)

$\{G$ has vertices $a = v_0, v_1, \ldots, v_n = z$ and lengths $w(v_i, v_j)$ where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in $G\}$

for $i = 1$ to n **do**
    $L(v_i) = \infty$
end for
$L(a) = 0$
$S = \emptyset$ {the labels are now initialized so that the label of a is 0 and all other labels are ∞, and S is the empty set }
while $z \notin S$ **do**
    $u =$ a vertex not in S with $L(u)$ is minimal
    $S = S \cup \{u\}$
    for all vertices v not in S **do**
        if $L(u) + w(u, v) < L(v)$ **then**
            $L(v) = L(u) + w(u, v)$
        end if
    end for
end while
return $L(z)\{L(z) =$ length of shortest path from a to $z\}$

Traveling Salesman

The traveling salesman problem asks for the circuit of minimum total weight in a weighted, complete, undirected graph that visits each vertex exactly once and returns to its starting point.

- Equivalent of asking for a Hamilton circuit with a minimum total weight in the complete graph.
- $\dfrac{(n-1)!}{2}$ circuits to examine
- This problem is NP-complete
- An approximation algorithm is used in practical approach

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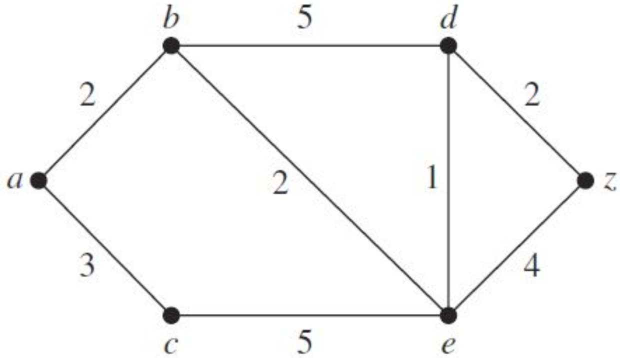
Find the length and shortest path between a and z in each of the weighted graphs

a)



Use Dijkstra's algorithm.

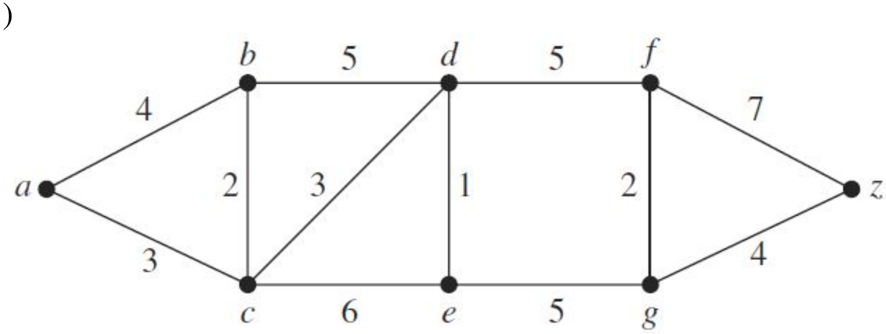
k	$L(a)$	$L(b)$	$L(c)$	$L(d)$	$L(e)$	$L(z)$	Vertex added to S
0	0	∞	∞	∞	∞	∞	a
1	0	2	3	∞	∞	∞	b
2	0	2	3	7	4	∞	c
3	0	2	3	7	4	∞	e
4	0	2	3	5	4	8	d
5	0	2	3	5	4	7	z

Prior vertex on shortest path to

k	b	c	d	e	z
1	a	a			
2			b	b	
3					
4			e		e
5					d

Our shortest path is a, b, e, d, z with length 7.

b)



k	$L(a)$	$L(b)$	$L(c)$	$L(d)$	$L(e)$	$L(f)$	$L(g)$	$L(z)$	Vertex added to S
0	0	∞	∞	∞	∞	∞	∞	∞	a
1	0	4	3	∞	∞	∞	∞	∞	c
2	0	4	3	6	9	∞	∞	∞	b
3	0	4	3	6	9	∞	∞	∞	d
4	0	4	3	6	7	11	∞	∞	e
5	0	4	3	6	7	11	12	∞	f
6	0	4	3	6	7	11	12	18	g
7	0	4	3	6	7	11	12	16	z

Prior vertex on shortest path to

k	b	c	d	e	f	g	z
1	a	a					
2			c	c			
3							
4				d	d		
5						e	
6							f
7							g

Our shortest path is a, c, d, e, g, z with length 16.