11.5 Minimum Spanning Trees

Minimum Spanning Tree

A *minimum spanning tree* in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.

Prim's Algorithm

An algorithm for finding a minimum spanning tree.

- Begin by choosing any edge with smallest weight, putting it into the spanning tree.
- Successively add to the tree edges of minimum weight that are incident to a vertex already in the tree, never forming a simple circuit with those edges already in the tree.
- Stop when $n - 1$ edges have been added.

Algorithm 3 *Prim*(G : weighted connected undirected graph with n vertices)

1: T = a minimum-weight edge
2: **for** i = 1 **to** $n - 2$ **do**
3: e = an edge of minimum weight incident to a vertex in T and not forming a simple circuit in T if added to T
4: $T = T$ with e added
5: **end for**
6: **return** T {T is a minimum spanning tree of G}

Kruskal's Algorithm

An algorithm for finding a minimum spanning tree.

- Begin by choosing an edge in the graph with minimum weight.
- Successively add edges with minimum weight that do not form a simple circuit with those edges already chosen.
- Stop after $n - 1$ edges have been selected.

Algorithm 4 *Kruskal*(G : weighted connected undirected graph with n vertices)

1: T = empty graph
2: **for** i = 1 **to** $n - 1$ **do**
3: e = any edge in G with smallest weight that does not form a simple circuit when added to T
4: $T = T$ with e added
5: **end for**
6: **return** T {T is a minimum spanning tree of G}

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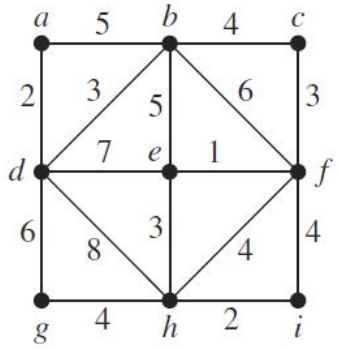
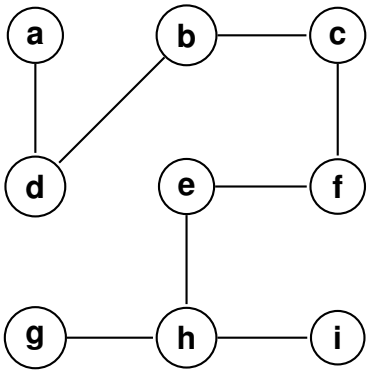
Use Prim's algorithm to find a minimum spanning tree for the given weighted graph.



Choice	Edge	Weight
1	$\{e, f\}$	1
2	$\{c, f\}$	3
3	$\{e, h\}$	3
4	$\{h, i\}$	2
5	$\{b, c\}$	4
6	$\{b, d\}$	3
7	$\{a, d\}$	2
8	$\{g, h\}$	4
		total: 22



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Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph in Exercise 3.

Choice	Edge	Weight
1	$\{e, f\}$	1
2	$\{a, d\}$	2
3	$\{h, i\}$	2
4	$\{b, d\}$	3
5	$\{c, f\}$	3
6	$\{e, h\}$	3
7	$\{b, c\}$	4
8	$\{g, h\}$	4
		total: 22