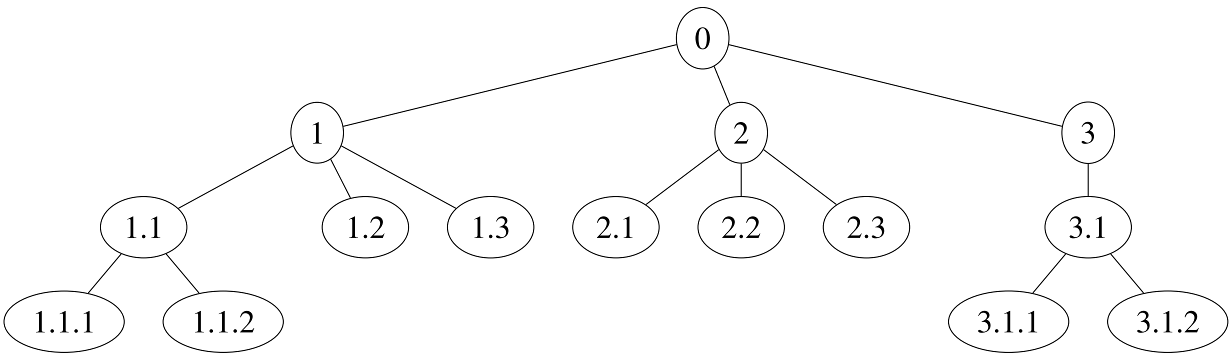
11.3 Tree Traversal

Universal Address Systems

A way to totally order the vertices of and ordered rooted tree. We do this recursively:

- Label the root with the integer 0. Then label its k children (at level 1) from left to right with $1, 2, 3, \ldots, k$.
- For each vertex v at level n with label A, label its k_v children, as they are drawn from left to right, with $A.1, A.2, \ldots, A.k_v$.



The lexicographic ordering for this tree is $0 < 1 < 1.1 < 1.1.1 < 1.1.2 < 1.2 < 1.3 < 2 < 2.1 < 2.2 < 2.3 < 3 < 3.1 < 3.1.1 < 3.1.2$

Traversal Algorithms

Preorder Traversal

Let T be an ordered rooted tree with root r. If T consists only of r, then r is the *preorder traversal* of T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right in T. The *preorder traversal* begins by visiting r. It continues by traversing T_1 in preorder, then T_2 in preorder, and so on, until T_n is traversed in preorder.

Algorithm $preorder(T$: ordered rooted tree)

r = root of T
list r
for each child c of r from left to right **do**
    $T(c)$ = subtree with c as its root
    $preorder(T(c))$
end for

Inorder Traversal

Let T be an ordered rooted tree with root r. If T consists only of r, then r is the *inorder traversal* of T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right. The *inorder traversal* begins by traversing T_1 in inorder, then visiting r. It continues by traversing T_2 in inorder, then T_3 in inorder, ..., and finally T_n in inorder.

Algorithm $inorder$(T : ordered rooted tree)

r = root of T
if r is a leaf **then**
    list r
else
    l = first child of r from left to right
    $T(l)$ = subtree with l as its root
    $inorder(T(l))$
    list r
    for each child c of r except for l from left to right **do**
        $T(c)$ = subtree with c as its root
        $inorder(T(c))$
    end for
end if

Postorder Traversal

Let T be an ordered rooted tree with root r. If T consists only of r, then r is the *postorder traversal* of T. Otherwise, suppose that $T_1, T_2, \dots, T_n$ are the subtrees at r from left to right. The *postorder traversal* begins by traversing T_1 in postorder, then T_2 in postorder, ..., then T_n in postorder, and ends by visiting r.

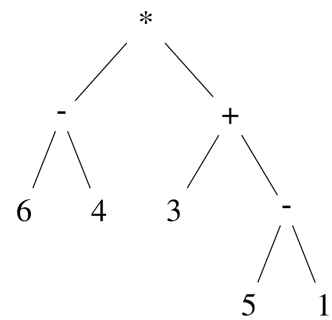
Algorithm $postorder$(T : ordered rooted tree)

r = root of T
for each child c of r from left to right **do**
    $T(c)$ = subtree with c as its root
    $postorder(T(c))$
end for
list r

Infix, Prefix, and Postfix Notation

Complicated expressions, such as compound propositions, combinations of sets, and arithmetic expressions can be represented by ordered root trees. For arithmetic expressions:

- Internal vertices: operations
- Leaves: variables or numbers



Infix Notation

Inorder traversal of the binary tree representing an expression produces the original expression with the elements and operations in the same order as they originally occurred, except unary operations. The fully parenthesized expression is the *infix form*.
Infix form: $(6 - 4) * (3 + (5 - 1))$

Prefix Notation

The *prefix form* of an expression can be obtained by traversing its rooted tree in preorder. Binary operator precedes its two operands. Evaluate an expression in prefix form by working right to left.
Prefix form: $* - 64 + 3 - 51$

Postfix Notation

The *postfix form* of an expression can be obtained by traversing its rooted tree in postorder. Binary operator follows its two operands. Evaluate an expression in postfix form by working left to right.
Postfix form: $64 - 351 - +*$

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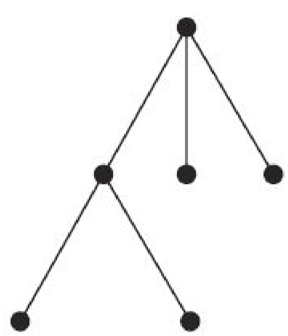
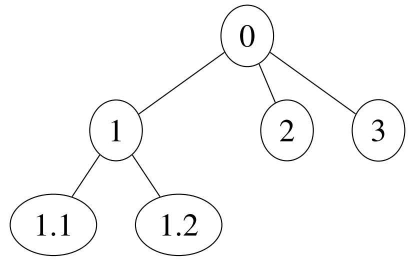
Construct the universal address system for the given ordered rooted tree. Then use this to order its vertices using the lexicographic order of their labels.





The lexicographic ordering is $0 < 1 < 1.1 < 1.2 < 2 < 3$.

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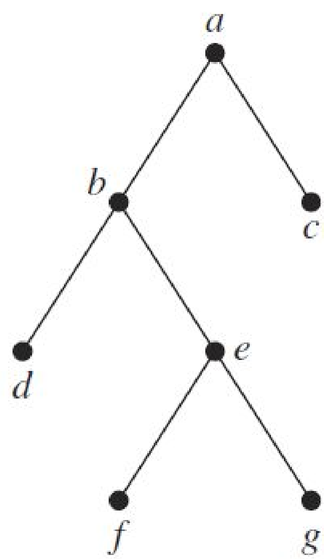
Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.



The preorder traversal is: a, b, d, e, f, g, c.

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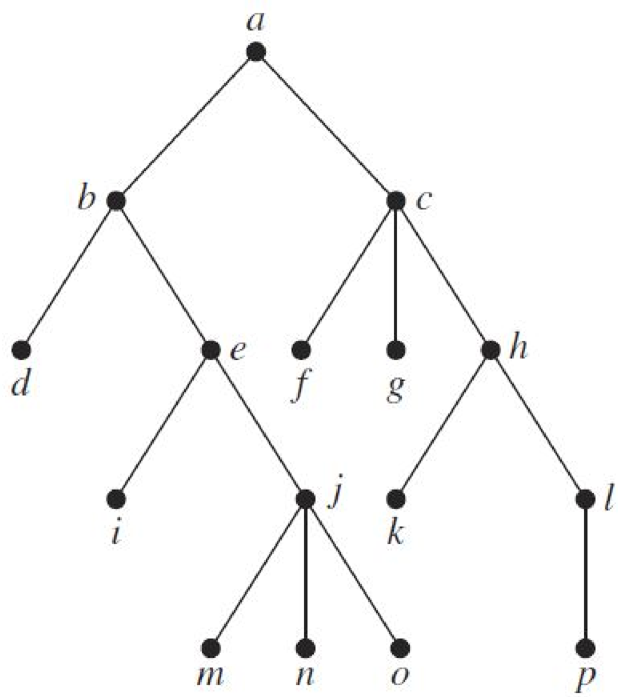
Determine the order in which an inorder traversal visits the vertices of the given ordered rooted tree.



The inorder traversal is: $d, b, i, e, m, j, n, o, a, f, c, g, k, h, p, l$.

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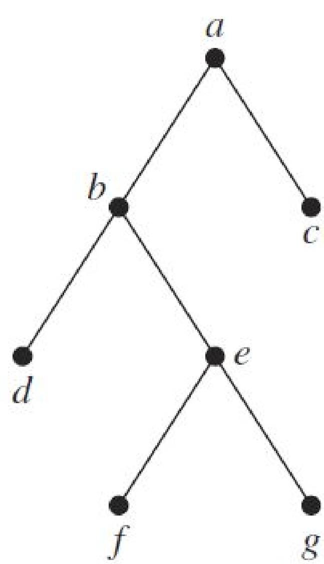
Determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.



The postorder traversal is: d, f, g, e, b, c, a

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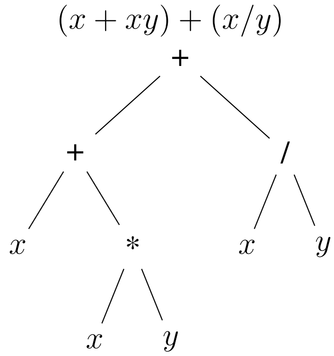
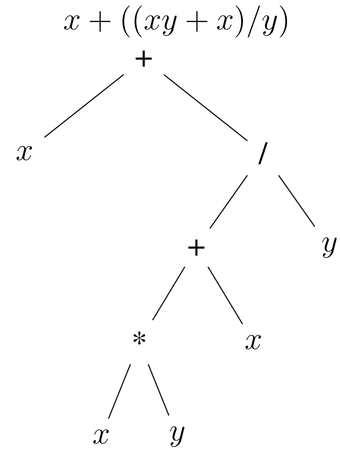
a) Represent the expressions $(x + xy) + (x/y)$ and $x + ((xy + x)/y)$ using binary trees.





b) Write these expressions in prefix notation.

$(x + xy) + (x/y)$ in prefix is: $+ + x * x\ y / x\ y$
$x + ((xy + x)/y)$ in prefix is: $+ x / + * x\ y\ x\ y$

c) Write these expressions in postfix notation.

$(x + xy) + (x/y)$ in postfix is: $x\ x\ y * + x\ y / +$
$x + ((xy + x)/y)$ in postfix is: $x\ x\ y * x + y / +$

d) Write these expressions in infix notation.

$(x + xy) + (x/y)$ in infix is: $((x + (x * y)) + (x/y))$
$x + ((xy + x)/y)$ in infix is: $(x + (((x * y) + x)/y))$

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What is the value of each of these prefix expressions?

a) $- * 2 / 8\ 4\ 3$

$- * 2 / 8\ 4\ 3$
$- * 2\ 2\ 3$
$-4\ 3$
1

b) $\uparrow - * 3\ 3 * 4\ 2\ 5$

$\uparrow - * 3\ 3 * 4\ 2\ 5$
$\uparrow - * 3\ 3\ 8\ 5$
$\uparrow -9\ 8\ 5$
$\uparrow 1\ 5$
1

c) $+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6 - 4\ 2$

$+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6 - 4\ 2$
$+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6\ 2$
$+ - \uparrow 3\ 2 \uparrow 2\ 3\ 3$
$+ - \uparrow 3\ 2\ 8\ 3$
$+ - 9\ 8\ 3$
$+1\ 3$
4

d) $* + 3 + 3 \uparrow 3 + 3\ 3\ 3$

$* + 3 + 3 \uparrow 3 + 3\ 3\ 3$
$* + 3 + 3 \uparrow 3\ 6\ 3$
$* + 3 + 3\ 729\ 3$
$* + 3\ 732\ 3$
$*735\ 3$
2205