## REs for Keywords

- It is easy to define a RE that describes all keywords
Key = 'if' | 'else' | 'for'| 'while' | 'int' | ..
- These can be split in groups if needed

Keyword = 'if' | 'else' | 'for' | ...
Type = 'int' | 'double'| |long' | ...

- The choice depends on what the next component (i.e., the parser) would like to see


## RE for Numbers

- Straightforward representation for integers
digits = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | ' 8 ' | ' 9 '
integer $=$ digits $^{+}$
- RE systems allow the use of '-' for ranges, sometimes with '[' and ']' digits $=[0-9]+$
- Floating point numbers are much more complicated
$2.00, .12 \mathrm{e}-12,312.00001 \mathrm{E}+12,4,3.141 \mathrm{e}-12$
- Here is one attempt

- Note the difference between meta-character and languagecharacters
'+' versus +, '-' versus -, '(' versus (, etc.
- Often books/documentations use different fonts for each level of language


## RE for Identifiers

- Here is a typical description
$\square$ letter $=a-z \mid A-Z$
$\square$ ident $=$ letter $\left(\right.$ letter $\mid$ digit | ' $\quad$ ') ${ }^{*}$
- Starts with a letter
- Has any number of letter or digit or '_' afterwards
- In C: ident = (letter | '_') (letter | digit | ' _') ${ }^{*}$


## RE for Phone Numbers

- Simple RE
$\square$ digit $=0-9$
$\square$ area $=$ digit digit digit
$\square$ exchange $=$ digit digit digit
$\square$ local = digit digit digit digit
$\square$ phonenumber = '(' area ')' ' '? exchange ('-'|' ') local
- The above describes the $10^{3+3+4}$ strings of the $L$ (phonenumber) language


## Regular Expression Practice

- Write regular expressions for
$\square$ All strings over alphabet $\{a, b, c\}$
- (a|b|c)*
- All strings over alphabet $\{a, b, c\}$ that contain substring 'abc' - (a|b|c)*abc(a|b|c)*
$\square$ All strings over alphabet $\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ that consist of one or more a's, followed by two b's, followed by whatever sequence of a's and c's
- $a^{+} \mathrm{bb}(\mathrm{a} \mid \mathrm{c})^{*}$
$\square$ All strings over alphabet $\{a, b, c\}$ such that they contain at least one of substrings 'abc' or 'cba'
- $\left((\mathrm{a}|\mathrm{b}| \mathrm{c})^{*} \mathrm{abc}(\mathrm{a}|\mathrm{b}| \mathrm{c})^{*} \mid(\mathrm{a}|\mathrm{b}| \mathrm{c})^{*} \mathrm{cba}(\mathrm{a}|\mathrm{b}| \mathrm{c})^{*}\right)$


## Regular Expression Practice

- Write regular expressions for
$\square$ All strings over alphabet $\{a, b, c\}$
$\square$ All strings over alphabet $\{a, b, c\}$ that contain substring 'abc'
$\square$ All strings over alphabet $\{a, b, c\}$ that consist of one or more a's, followed by two b's, followed by whatever sequence of a's and c's
$\square$ All strings over alphabet $\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ such that they contain at least one of substrings 'abc' or 'cba'


## Automaton Examples



- This automaton accepts input 'if'


## Automaton Examples



- This automaton accepts strings that start with a 0 , then have any number of 1's, and end with a 0
- Note the natural correspondence between automata and REs: $01^{*} 0$
- Question: can we represent all REs with simple automata?
- Answer: yes
- Therefore, if we write a piece of code that implements arbitrary automata, we have a piece of code that implements arbitrary REs, and we have a lexer!

Not _this_ simple, but close

## Example REs and DFA

- Say we want to represent RE 'a*b*c*d*e' with aDFA



## Example REs and NFA

- ' $a^{+} b^{+} c^{+} d^{+} e^{\prime}$ : easy modification

- But now we have multiple choices for a given character at each state!
$\square$ e.g., two 'a' arrows leaving n


## Automaton vs. RE Practice

- Write REs for the following NFAs



## A Grammar for Expressions

| Expr | $\rightarrow$ Expr Op Expr |
| :--- | :--- |
| Expr | $\rightarrow$ Number \| Identifier |
| Identifier | $\rightarrow$ Letter \| Letter Identifier |
| Letter | $\rightarrow$ a-z |
| Op | $\rightarrow$ "+" ""-" \|"*" |""" |
| Number | $\rightarrow$ Digit Number \| Digit |
| Digit | $\rightarrow 0\|1\| 2\|3\| 4\|5\| 6\|7\| 8 \mid 9$ |

Expr $\rightarrow$ Expr Op Expr $\rightarrow$ Number Op Expr $\rightarrow$
Digit Number Op Expr $\rightarrow 3$ Number Op Expr $\rightarrow 34$ Op Expr $\rightarrow$ 34 * Expr $\rightarrow 34$ * Identifier $\rightarrow 34$ * Letter Identifier $\rightarrow$
34 * a Identifier $\rightarrow 34$ * a Letter $\rightarrow 34$ * ax

## Automaton vs. RE Practice

- Write REs for the following NFAs


```
                                    a*b*(ab*a | bab+)
```


## Derivations as Trees

- A convenient and natural way to represent a sequence of derivations is a syntactic tree or parse tree
- Example: Expr $\rightarrow$ Expr Op Expr $\rightarrow$ Number Op Expr $\rightarrow$ Digit Number Op Expr $\rightarrow 3$ Number Op Expr $\rightarrow 34$ Op Expr $\rightarrow 34$ * Expr $\rightarrow 34$ * Identifier $\rightarrow$ $34^{*}$ Letter Identifier $\rightarrow 34^{*}$ a Identifier $\rightarrow 34$ * a Letter $\rightarrow 34^{*}$ ax



## Derivations as Trees

- In the parser, derivations are implemented as trees
- Often, we draw trees without the full derivations
- Example:



## Grammar Practice

- Consider the CFG:

$$
\begin{array}{lll}
\mathrm{S} & \rightarrow(\mathrm{~L}) \mid a \\
\mathrm{~L} & \rightarrow \mathrm{~L}, \mathrm{~S} \mid \mathrm{S}
\end{array}
$$

Draw parse trees for:
(a, a)
(a, ((a, a), (a, a)))


## Grammar Practice

- Consider the CFG:

$$
\begin{array}{ll}
S & \rightarrow(L) \mid a \\
L & \rightarrow L, S \mid S
\end{array}
$$

Draw parse trees for:
(a, a)
(a, ((a, a), (a, a)))

## Grammar Practice

- Consider the CFG:

$$
\begin{array}{ll}
S & \rightarrow(L) \mid a \\
L & \rightarrow L, S \mid S
\end{array}
$$

Draw parse trees for:
(a, a)
(a, ((a, a), (a, a)))


## Grammar Practice

- Write a CFG for the language of well-formed parenthesized expressions
$\square(),(()),()(),(()())$, etc.: OK
$\square())),(,,(()),,((($, etc.: not OK


## Grammar Practice

- Is the following grammar ambiguous?
$\mathrm{A} \rightarrow \mathrm{A}$ "and" $\mathrm{A}|" n o t " \mathrm{~A}| " 0$ |" 1 "


## Grammar Practice

- Write a CFG for the language of well-formed parenthesized expressions
$\square(),(()),()(),(()())$, etc.: OK
$\square())),(,,(()),,((($, etc.: not OK

```
P->() | PP | (P)
```


## Grammar Practice

- Is the following grammar ambiguous?
$\mathrm{A} \rightarrow \mathrm{A}$ "and" $\mathrm{A}|\operatorname{not} \mathrm{A}| 0 \mid 1$


Yes!


