Numbers and Computers: Examples and Sample Problemsm

ICS312
Machine-Level and Systems Programming

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Conversions

- What is 52_{10} in binary?

52_{10} in binary

- Systematic method:
  - $52 = 26*2 + 0$
  - $26 = 13*2 + 0$
  - $13 = 6*2 + 1$
  - $6 = 3*2 + 0$
  - $3 = 1*2 + 1$
  - $1 = 0*2 + 1$
  - Answer: 110100
- Intuitive method (for "small" numbers)
  - 52 is lower than 64, so it's 32 + some other powers of 2
  - 32 + 16 is 48, so 52 is 32 + 16 + some other powers of 2
  - 52 - 48 = 4, so we have: 52 = 32 + 16 + 4
  - Therefore: 110100
    - We have 64, 32, not 16, not 8, 4, not 2, not 1

Conversions

- What is 2049_{10} in binary?

2049_{10} in binary

- The systematic method is really long here
  - simple though, but tedious
- It's easier to see that 2049_{10} is $2048_{10} + 1_{10}$
  - 2048_{10} is $2^{11} = 100000000000_2$
  - 1 is $2^0 = 1_2$
- Therefore
  - $249_{10} = 100000000001_2$

- In general, one likes to find our "nearby" powers of 2

Conversions

- What is 1021_{10} in binary?

1021_{10} in binary

- This is "close to" 1024_{10}
- We know that 1024_{10} is 1000000000_2
- More useful: 1023_{10} is 111111111_2
- So we can "count backwards"
- 111111110_2 comes before 111111111_2, and therefore it is 1022_{10}
- 111111101_2 comes before 111111110_2, and therefore it is 1021_{10}
- Answer: 111111101_2

Conversions

- What is $B8_{16}$ in binary?

B8$_{16}$ in binary

- Just "glue" the 2 4-bit conversions together
  - B$_{16}$ = 1011$_2$, 8$_{16}$ = 1000$_2$
  - Answer: 10111000

- How do I know that B$_{16}$ = 1011$_2$?
  - Just go back to decimal
    - B$_{16}$ = 11$_{10}$
    - 11$_{10}$ = 1011$_2$

Conticals

Conversions

- What is 51_{10} in hexadecimal?
- What is 0110_2 in hexadecimal?

Solutions

- What is 51_{10} in hexadecimal?
  - $51 = 3*16 + 3$
  - Answer: 33
- What is 0110_2 in hexadecimal?
  - $0110_2 = 6_{10} = 6_{16}$
  - Answer: 6

More Conversions

- What is 123_{10} in binary?

- What is $F3EA_{16}$ in binary?

- What is 111_{10} in hexadecimal?

- What is 100110_2 in hexadecimal?

Solutions

■ Conversions:

  ☐ What is 123_{10} in binary?

    1111011   (127 - 4)

  ☐ What is $F3EA_{16}$ in binary?

    1111001111101010

  ☐ What is 111_{10} in hexadecimal?

    6F   (112 -1)

  ☐ What is 100110_2 in hexadecimal?

    26

Always try to find simple "tricks" if you can

Binary addition

- What is: 10101101 + 11001011 ?

Solution

- What is: 10101101 + 11001011 ?

$$
\begin{array}{r}
\textcolor{red}{c}\quad\ \textcolor{red}{c\,c\,c} \\
10101101 \\
+\ 11001011 \\
\hline
=\ \ 101111000
\end{array}
$$

Hex addition

- What is: A5F + E32 ?

Solution

- What is: A5F + E32 ?

$$
\begin{array}{r}
\textcolor{red}{c}\ \ \textcolor{red}{c} \qquad\quad \\
\texttt{A5F} \\
+ \qquad \texttt{E32} \\
= \qquad \texttt{1891}
\end{array}
$$

- Small "trick": adding F to a digit takes that digit 1 lower and generates a carry
  - F + 7 = 6 and a carry
  - F + E = D and a carry

Another binary addition

- What is 1010111 + 1110111?

Solution

- What is 1010111 + 1110111?

<pre>
      ccc ccc
       1010111
+   1110111
=   11001110
</pre>

Another hex addition

- What is AF3F + EE8D?

Solution

- What is AF3F + EE8D?

$$
\begin{array}{r}
\texttt{\textcolor{red}{c} \ \textcolor{red}{c}} \\
\texttt{AF3F} \\
+ \quad \texttt{EE8D} \\
= \texttt{19DCC}
\end{array}
$$

Two's complement

- What is the 2's complement 2-byte representation of -153_{10} in hexadecimal?

Solution

- What is the 2's complement representation of -153_{10} in hexadecimal?

  - $153_{10} = 0099_{16}$

  - complement: FF66

  - add 1: **FF67**

Two's complement

- What is the decimal value of FF4A, a 2-byte numbers stored in 2's complement fashion?

Solution

- What is the decimal value of FF4A, a 2-byte numbers stored in 2's complement fashion?
  - FF4A = $1\ldots._2$
  - Therefore it represents a negative number, let's invert it
  - Invert: 00B5
  - Add 1: 00B6 = B6
  - $B6_{16} = 11*16 + 6 = 176 + 6 = 182_{10}$
  - Therefore, in 2's complement notation, FF4A is -182_{10}

Two's complement

- What is the 2's complement 1-byte representation of -81_{10} in hexadecimal?

Solution

- What is the 2's complement 1-byte representation of -81_{10} in hexadecimal?
  - $81_{10} = 51_{16}$
  - complement: AE
  - add 1: AF

Two's complement

- What is the decimal value of 76h, a 1-byte number stored in 2's complement fashion?

Solution

- What is the decimal value of 76, a 1-byte numbers stored in 2's complement fashion?
  - It's a positive number, so 76 is simply the hex value of the integer
  - Answer: $7*16^1 + 6*16^0 = 118_{10}$

Ranges of numbers

- What is the largest **unsigned** decimal number that can be encoded with 8 bits?
- What is the smallest **unsigned** decimal number that can be encoded with 8 bits?
- What is the largest **signed** decimal number that can be encoded with 8 bits?
- What is the smallest **signed** decimal number that can be encoded with 8 bits?
- What is the 2's complement representation of -1_{10} with 32 bits?

Solutions

- What is the largest **unsigned** decimal number that can be encoded with 8 bits?
  - 255 (i.e., FF in 2's complement representation)
- What is the smallest **unsigned** decimal number that can be encoded with 8 bits?
  - 0 (i.e., 00 in 2's complement representation)
- What is the largest **signed** decimal number that can be encoded with 8 bits?
  - Largest that isn't negative: 7F in 2's complement representation = 127_{10}
- What is the smallest **signed** decimal number that can be encoded with 8 bits?
  - Smallest that isn't positive: 80 in 2's complement representation = -128_{10}
- What is the 2's complement representation of -1_{10} with 32 bits?
  - 1 = 00000001; complement: FFFFFFFE; add one: FFFFFFFF